Some basic knowledge about a transmission line
worth knowing when dealing with
VNWA calibration standards

Preface:

Prof. Dr.-Ing Jörg Hugel has been so kind to prepare some notes to me about transmission line mathematics related to the understanding of calibration standards. He also introduced me to the simulation programs Quickfield and FEMM

I want to share with the VNWA lovers these formulas and have added some details and comments.

Dan Maguire AC6AL, the author of the fantastic program ZPlots, which seamless integrates with the VNWA software, is also an excellent source of transmission line mathematics, and his homepage is very useful in the said context
http://www.ac6la.com/

The basic definitions:

The speed **v**, also expressed as the length divided by the delay **t**, and characteristic impedance Z_0 of a coaxial conductor are with following relations to the distributed serial $\overline{L}$ and shunt $\overline{C}$ per length unit of the coaxial conductor

$$v = \frac{1}{\sqrt{\overline{L} \cdot \overline{C}}} = \frac{l}{t} \quad \text{and} \quad Z_0 = \sqrt{\frac{\overline{L}}{\overline{C}}}$$

If we multiply v and Z_0 we get the following result:

$\frac{l}{t} \cdot Z_0 = \frac{1}{\overline{C}} = \frac{l}{C}$ by re-arranging the formula we get:

$\boldsymbol{t = Z_0 \cdot C}$ which means delay t is $\boldsymbol{Z_0}$ multiplied with the capacitance for the **physical length l** of the conductor

With Z_0=50 ohm we get

$\boldsymbol{t = 0.05 \cdot C}$ where t is expressed in ps (pico seconds) and C expressed in fF (femto Farad)

As an example for $\boldsymbol{C = \text{20fF we get } t = \text{1ps}}$ or **1fF = 0.05ps**

<u>If we want to solve with respect to the inductance L then we can do the following</u>:

The expression $Z_0 = \sqrt{\frac{\overline{L}}{\overline{C}}}$ we square to $Z_0^2 = \frac{\overline{L}}{\overline{C}} \gg \overline{C} = \frac{L}{Z_0^2} \gg C = \frac{L}{Z_0^2}$ and from above we know $t = Z_0 \cdot C$ then

$\boldsymbol{t = \frac{L}{Z_0}}$ which means delay t is Z_0 divided into the inductance L for the physical length l of the conductor

With $Z_{0 =}$ 50 ohm we get

$\boldsymbol{t = 0.02 \cdot L}$ where t is expressed in ps (pico seconds) and L expressed in pH (pico Henry)

As an example for $\boldsymbol{C = \text{50pH}}$ we get $\boldsymbol{t = \text{1ps}}$ or **1pH = 0.02ps**

Another convenient expression is the $\frac{L}{C}$ ratio equal to Z_0^2 as when C is found the L can be calculated.

$$\boldsymbol{L = C \cdot Z_0^2}$$

The only additional mathematics we need is calculation of Z_0 based on diameter of the coaxial conductor inner and outer diameter. I use on my iPhone the Huber Suhner app called RF Tools or you may use online tools like http://www1.sphere.ne.jp/i-lab/ilab/tool/cx_line_e.htm or http://www.microwaves101.com/calculators/863-coax-calculator if you prefer inches or
http://www.pasternack.com/t-calculator-coax-cutoff.aspx which is a mix of mm and inches

In relationship to calibration standards these details are needed to understand if you are designing your own calibration standards using standard adaptor parts. However some further details are handssome to study regarding formulas for a coaxial conductors. The homepage http://www.microwaves101.com is an excellent source of such information and just search under T for Transmission lines and we get all the important definitions presented at:

and if selecting coax overview we find:

Coax Cross-section
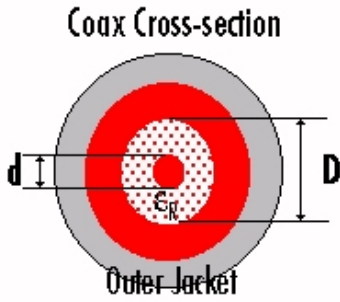
$$C = 2\pi \cdot \varepsilon_0 \cdot \frac{\varepsilon_r}{\ln\left(\frac{D}{d}\right)} \; \text{Farads/meter where } \varepsilon_0 = 8.854 \cdot 10^{-12} \; \text{Farads/meter}$$

$$L = \frac{\mu_0}{2\pi} \cdot \mu_r \cdot \ln\left(\frac{D}{d}\right) \; \text{Henry/meter where } \mu_0 = 4\pi \cdot 10^{-7} \; \text{Henry/meter}$$

As μ_r *as* usual is = 1 for all dielectrics we can simplify the expressions to

$$\boldsymbol{C = 55.63132 \cdot \frac{\varepsilon_r}{\ln\left(\frac{D}{d}\right)}} \quad \text{or} \quad \boldsymbol{C = 24.16037 \cdot \frac{\varepsilon_r}{\log\left(\frac{D}{d}\right)}} \; \textbf{\textit{pF/meter}}$$

$$\boldsymbol{L = 200 \cdot \ln\left(\frac{D}{d}\right)} \quad \text{or} \quad \boldsymbol{L = 460.517 \cdot \log\left(\frac{D}{d}\right)} \; \textbf{\textit{nH/meter}}$$

As $\boldsymbol{Z_0 = \sqrt{\dfrac{L}{C}}}$ then we have $\quad \boldsymbol{Z_0 = \dfrac{59.95912}{\sqrt{\varepsilon_r}} \cdot \ln\left(\dfrac{D}{d}\right)} \quad$ or $\quad \boldsymbol{Z_0 = \dfrac{138.0609}{\sqrt{\varepsilon_r}} \cdot \log\left(\dfrac{D}{d}\right)}$

Note: $\boldsymbol{\varepsilon_r = 8.854}$ *is the Permitivity of free space* and $\boldsymbol{\mu_0 = 4\pi \cdot 10^{-7}}$*is the Permeability of free space*

One final detail: $\quad \boldsymbol{v_{light} = c = 1/\sqrt{\varepsilon_0 \cdot \mu_0}}$ **and** we get $\boldsymbol{c = 299.7956}$*meters/second*

In the next section these calculation of **Z_0 C and L** plays an important role, because you cannot measure the delay of an adaptor not being exact 50 ohm impedance with an VNWA (or any other VNA) being calibrated to 50 ohm, as if the impedance is lower than 50 ohm the measured delay will longer than the physical delay and vice versa for an adaptor with a higher impedance than 50 ohm.
The only way to overcome this problem is by modelling the adaptor in the VNWA calibration setting by using the arbitrary calibration method and the calculated Z_0 C and L values.

There is however one more effect to take into account in some cases and that is the delay characteristic impedance of an coaxial conductor is frequency dependent at low frequencies, an often overlooked phenomena.

If we consider the formula for Z_0 and include the looses $Z_{lossy} = \sqrt{\dfrac{r+j\omega L}{g+j\omega C}}$ then then conductivity g is negligible and

the $Z_{lossy} = \sqrt{\dfrac{r+j\omega L}{j\omega C}}$ only has the losses due to resistance per length unit and indicates the impedance must

increases when frequency is lowered. If we take some realistic data from e.g. a RG6 75 ohm cable, then it has per feet C=16.2pf, L=91.125nH and r=6.4 mohm. Creating a custom trace model in the VNWA we see the following.
It is obvious that if we have a number of contact resistances coupling adaptor together this phenomena escalates.
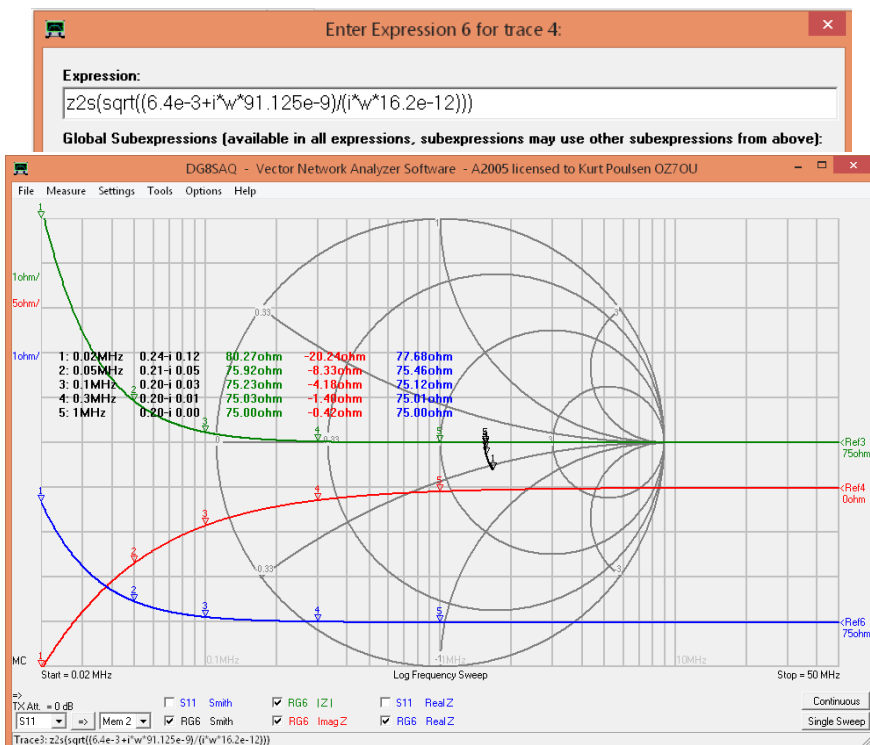
**A practical method how to investigate if a calibration is perfect:**
It is always difficult to judge if a calibration is perfect and you may use a T-Check which is time consuming but it includes both S11, S22, S21 and S12 calbration. However if you only need to test a calibration for S11 measurement then using an about **45cm long open** semi rigid cable for frequencies up to 500MHz and shorter for higher frequencies e.g. **18cm to 1300MHz**, and perform a sweep then the trace must run along the circumference of the Smith Chart and slightly moves inward as frequency increases due to cable losses.

A small collection of my semirigid cables ☺

The trace might look like this which is not OK.
Please enable a S11 dB trace with e.g. 0.2dB per division (the first red trace) which is oscillating across the frequency range.(See the arrow)
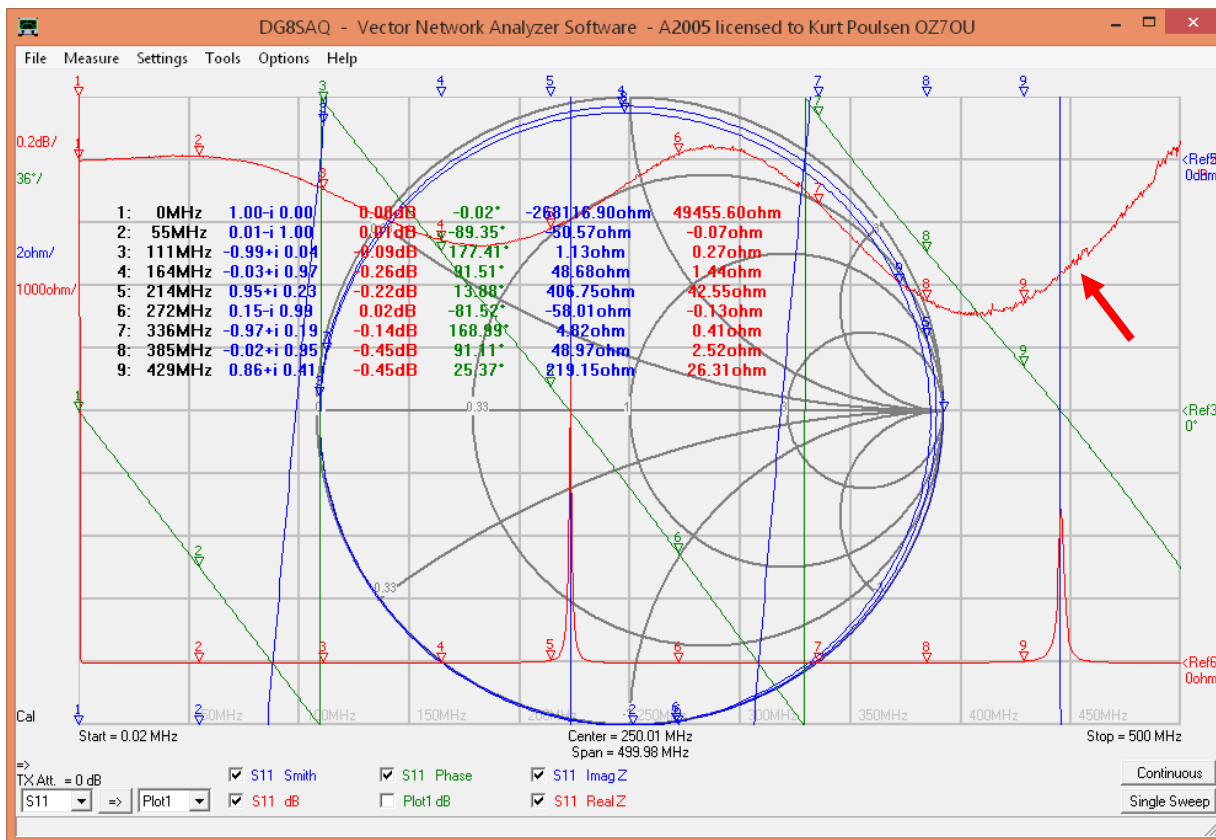
 Providing your short and open calibration standards are well defined (like the Rosenberger kit from SDR-KITS) incl. the precise value of the load resistance (which is measured by SDR down to 0.01ohm accuracy) there is only one source for fine tuning and that is the **fringe C of the load** which varies from one example to another.

Enable realtime recalibration feature with a tickmark
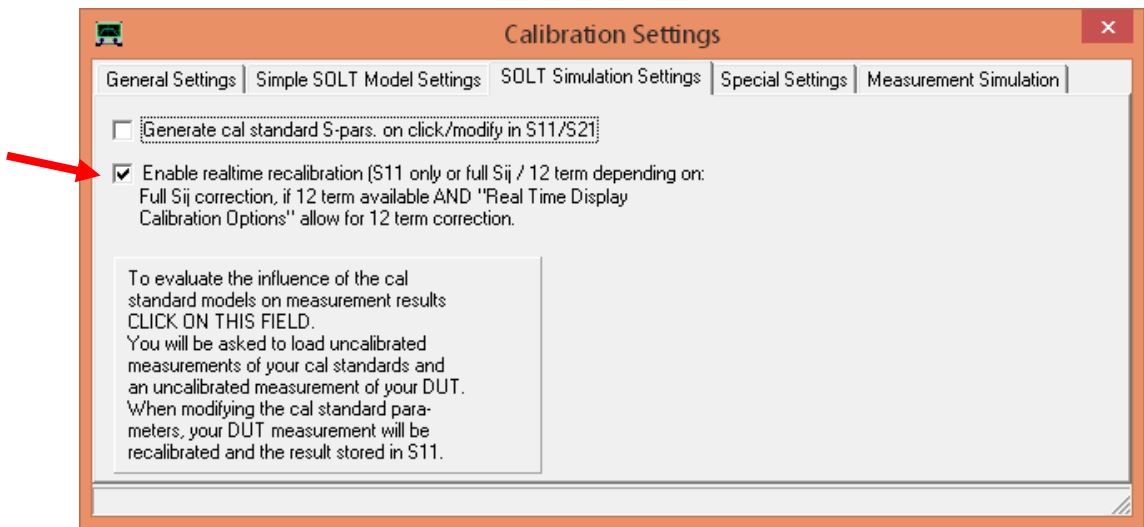
Open the calibration settings and position you mouse in the C|| field a scroll with the mouse wheel **until the S11 dB trace is completely flat** above 100MHz just as seen below for the red trace.
Then disable the realtime recalibration and save you calibration file again. Now you have perfect calibration.
Please note the open settings interact with the C|| setting so if you for any reason change the open setting repeat the process for C||.
(The value -145 indicates that fringe is inductive and is derived from a homemade UHF calibration kit)
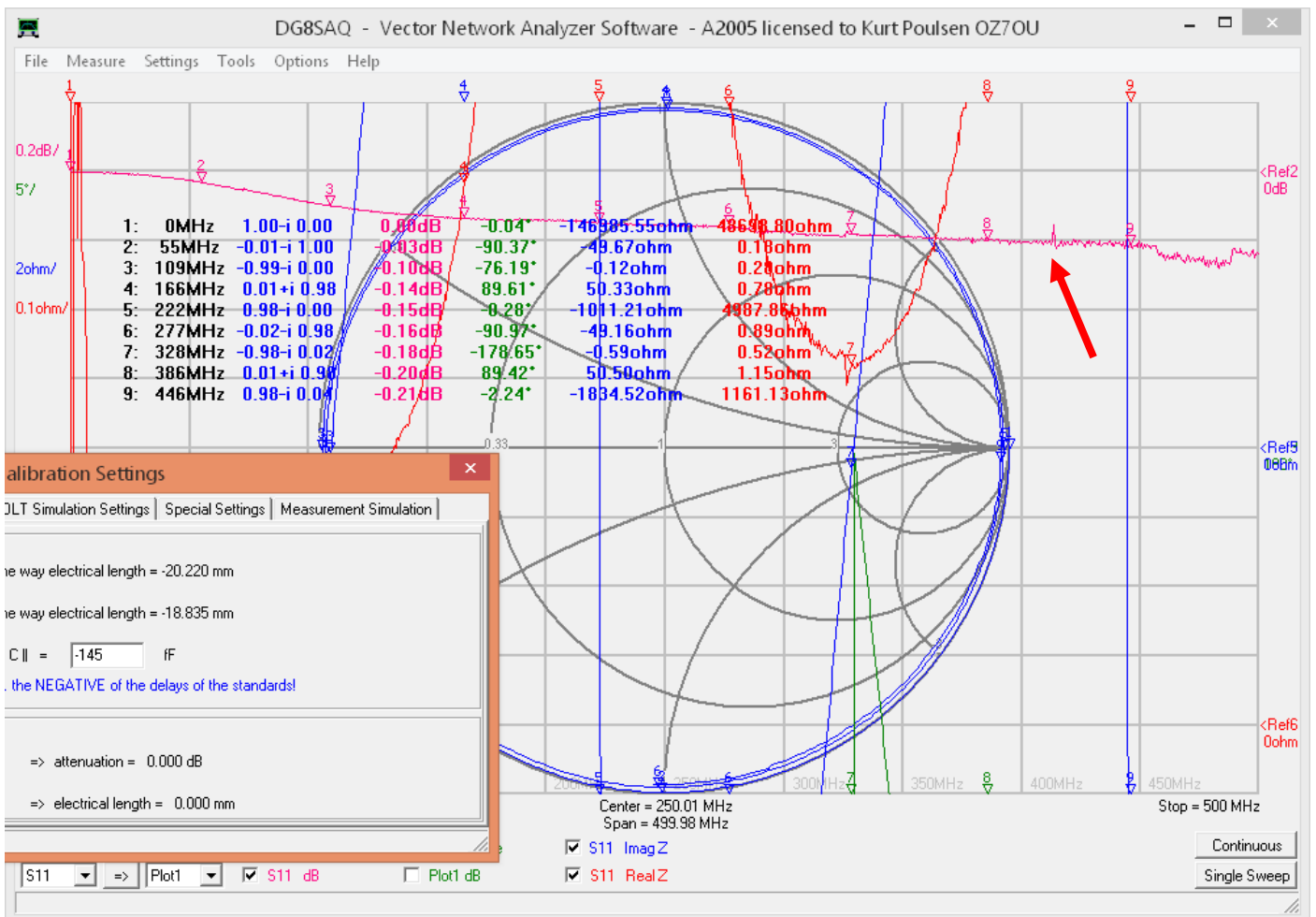
Next section is just notes about some practical details how above mathematics and methods was used in the process for a UHF/PL259 female short and open calibration standard.
Consider everything below as just a clarification appendix to the primary content of this report and solely done for my own purposes, experimenting with the Quickfield, FEMM and atlc2 simulation programs.

Avoid e.g. for a UHF/PL259 calibration kit, the use of isolation material in the adaptors and make the calibration standards mechanically as simple as possible. Use straight adaptors sections with a plane shorting disk for female short and a section of empty straight connector housing with a center conduction extender flush with the end of the empty straight section of adaptor housing for e.g female open.

Below to the left is an example of my UHF female open calibration standard and to the right the female short calibration standard before I soldered the shorting disk to the adaptor housing.

The center conductor extender has two diameters, as being one half of the center conductor for a female female adaptor but that is not a problem because a caliper and the free simulation software FEMM is all you need to calculate the C value, both with and without the endpoint radiation/fringe capacitance.

The internal depth above the calibration plane determines the delay, which always is independent of the actual nominal Z_0, and for an airline it is the length in mm divided by 0.3 with result in ps.

(If being pedantic the division should be with 0.2997956)

Then the nominal Z_0 can be calculated based on the delay and the FEMM simulated C value (without end point radiation) by the formula $t = Z_0 \cdot C$ (Z_0 the nominal just calculated Z_0 not 50 ohm). You may check the value by an estimated calculation of Z_0 based on center conductor diameter and internal diameter observing that the center conductor extender is not one diameter only and a small difference exist.

The next step is a second simulation with FEMM to determine the C value inclusive the endpoint radiation which is added to the C value in the arbitrary calibration setting model, whereas the L value maintained for the model. Thus we have included the effect of the endpoint radiation/fringe capacitance which lowers the Z_0 a bit. A very accurate compensated delay is found for the open standard.

The accuracy depends solely how accurate you model in the FEMM so you may consult a manufacturers drawings to find how deep the isolation is recessed behind the reference plane and the center pin length of the mating male adaptor a we must use dimension measured from the calibration plane outwards.

Similar techniques are used for the female short which is simpler as not having a fringe component to take care of.

The nominel Z_0 calculated based on the physical dimension measured with the caliper and the delay in ps calculated based on the depth in mm divided by 0.3. Based on the Z_0 found and the delay t calculated we find the L and C based on the formulas $C = \frac{t}{Z_0}$ and $L = Z_0 \cdot t$ which we then using for modeling the short in the arbitrary calibration settings.

For a more detailed study I refer to my complete document about making my UHF/PL259 male and female calibration kit incl. determination of the delays of thru adaptors for S21 calibration at following link:

http://www.hamcom.dk/VNWA/SOT259 calibration standards.pdf

Some notes about FEMM and another simulation program called atlc2

For finding the C value the free field simulation program called FEMM http://www.femm.info/wiki/download is very accurate and below examples demonstrate the calculation of the delay for a 30 ohm and a 50 ohm transmission line which always will have the same physical delay.

 The exercise below is only to investigate accuracy comparisons between the two applications.

 As stated, these delay calculations are based on a coaxial structure of exactly 50 ohm and what happens if the Z_0 differs from 50 ohm and we measure the coaxial structure e.g. a thru adaptor, with a VNWA calibrated to 50 ohm with a precise 50 ohm short, open and load calibration kit. ? As just said they will have the same physical delay, but when delay measured in a 50 calibrated environment (or any other for that matter) they will differ as the L/C ratio is not correct for the 30 ohm part, which the VNWA assumes is a 50 ohm ratio when measuring the two delays.
Let us take two cases being nom. Z_0 of 30 ohm and 50ohm. For a given length l the C value increases when Z_0 is lowered so what about the delay ? as stated to be the same. We can examine how accurate FEMM calculates
For a 50 ohm airline the D/d ratio 2.3 is and for 30 ohm 1.65 with fixed inside diameter D or othervise with D=12 having a center conductor diameter of 5.215178502818858 mm for 50 ohm and 7.278367917166163 mm for 30 ohm ☺. We use 5.22mm for 50 ohm and 7.28 mm for 30 ohm.

A 20mmm long TL (Transmission Line) with D=12 has a C value for 30 ohm of 2.22501pF and 1.33711pF for 50 ohm according to a FEMM simulation.
Delay thus 0.03*2225.01fF for 30 ohm = 66.7503ps. For 50 ohm 0.05*1337.11fF = 66.8555ps and thus in reality identical.
The very small deviation which is pure theoretical can solely be due to D/d ratio's with only 2 digits resolution.
If we use the $\frac{L}{C} \; ratio \; = \; Z_0^2$ then L calculated to 2.002509nH for 30 ohm and 3.342775nH for 50 ohm
The coaxial line calculator says 111.112 pF/m - 99.999nH/m for 30 ohm and 66.667pF/m - 166.667nH/m for 50 ohm.
Converted to 20mm length they are 2.2224pF – 1.99998nH for 30 ohm and 1.33334pF – 3.33334nH for 50 ohm which indicate same result with narrow tolerances.

Another simulation software is called atlc2 from http://www.hdtvprimer.com/kq6qv/atlc2.html which does a similar job, and can add frequency and skin depth parameters.
Running an atlc2 simulation for 50 ohm the results are for 1 meter length of TL:

Please "solve fully" and when determining C and L use the results either "C and vf results" or L and Rs results" which come closest to the Z_0. For below it is L and Rs results which is the correct one to use.
**Disable "Restrict to skin depth" and remark dimensions are in inch !!**

C 66.794pf or 1.33588pF for 20mm pretty identical to the FEMM simulation and leads to a delay of 66.794ps .
L calculated to 0.1666uh / 100 cm or 0.1666nH per mm which for 20mm provides 3.332nH   leading to a delay of 66.64ps so also for this atlc2 application is fairly accurate results compared to FEMM and a much faster method.
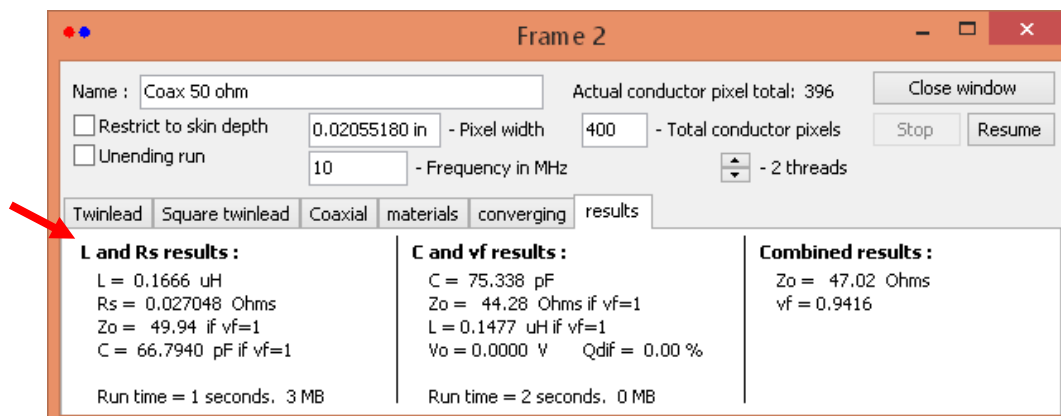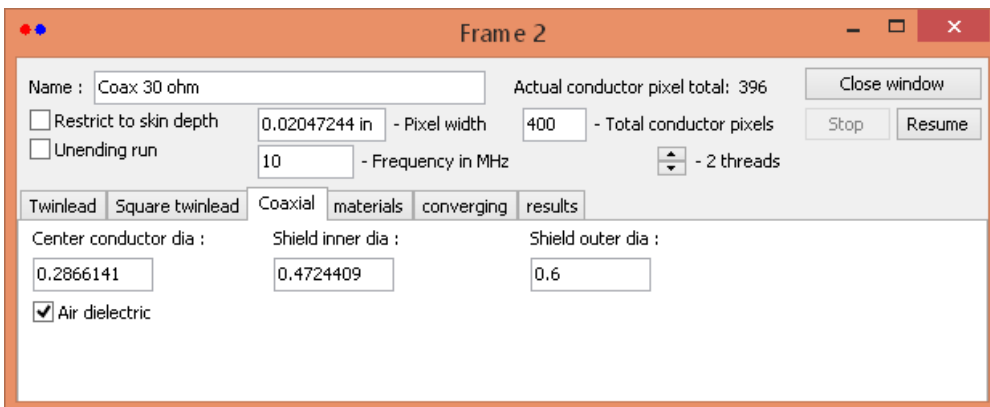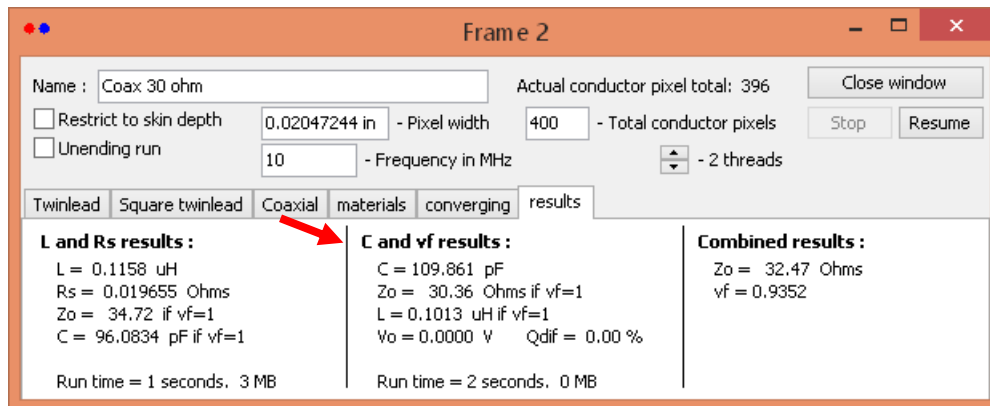
The initial settings seen below.
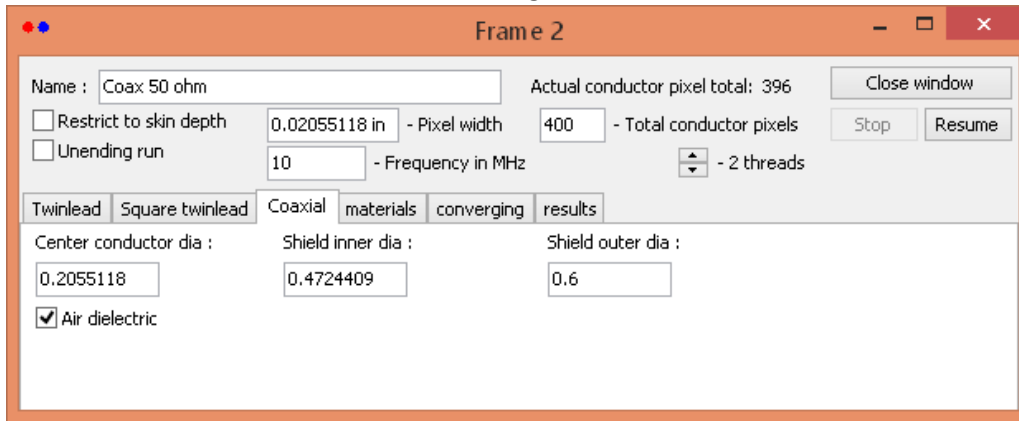
For a 30 ohm TL the following result when running a atlc2 simulation results are:
C 109.861pF or 2.1972pF for 20mm identical to the FEMM simulation leading to a delay of 65.91ps.
 L calculated to 0.1013uh / 100 cm or 0.1013nH per mm, which for 20mm is 2.026nH and delay 67.533ps.

Summary and comparison:

The calculation and simulation with FEMM and atlc2 leads to:

For 30 ohm the delay based on C thus 0.03*2197.2 fF = 65.91ps with 66.7503ps for FEMM simulation (1.3%)

For 50 ohm 0.05*1335.88fF = 66.794ps   with 66.8555ps for FEM simulation (0.09%)

For 30 ohm the L value calculated to 2.026nH / delay 67.533ps. FEMM calculated to 2.002509nH / 66.7503ps (-1.2%)

For 50 ohm the L value calculated to 3.332nH / delay 66.64ps. FEMM calculated to 3.342775nH / 66.8555ps (0.3%)

Conclusion:

These calculations demonstrates that the atlc2 is well suited and accurate enough to determine the C and L values to be used in the arbitrary calibration model settings.

The only requirement is the internal diameter and the centerpin diameter can be measured accurately and are as straight as possible so the atlc2 calculations can be accurate.

My recommendation is to use the FEMM in favor of atlc2 because with FEMM you can model the adaptor very precisely with the modeling editor whereas atlc2 is bases on uniform transmission lines.

FEMM obtained from the link: http://www.femm.info/wiki/download   http://www.femm.info/wiki/HomePage

Atlc2 obtained from the link: http://www.hdtvprimer.com/kq6qv/atlc2.html which is a windows version made by Ken Nist KG6QV and which is inspired by the original atlc program developed by Dr. David Kirkby G8WRB

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Kurt Poulsen de OZ7OU